Miscellaneous Definitions and Formulas

Imaginary Unit

$$j = \sqrt{-1}$$

Hyperbolic Functions

$$\sinh x = \frac{e^x - e^{-x}}{2}$$

$$\cosh x = \frac{e^x + e^{-x}}{2}$$

$$\tanh x = \frac{\sinh x}{\cosh x} = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

$$\coth x = \frac{\cosh x}{\sinh x} = \frac{e^x + e^{-x}}{e^x - e^{-x}}$$

$$\operatorname{sech} x = \frac{1}{\cosh x} = \frac{2}{e^x + e^{-x}}$$

$$\operatorname{csch} x = \frac{1}{\sinh x} = \frac{2}{e^x - e^{-x}}$$

Relationships Between Trigonometric and Hyperbolic Functions

$$\sinh x = -j\sin(jx)$$

$$\cosh x = \cos(jx)$$

$$\tanh x = -j\tan(jx)$$

$$\coth x = j\cot(jx)$$

$$\operatorname{sech} x = \sec(jx)$$

$$\operatorname{csch} x = j \csc(jx)$$

Fourier Series

Trigonometric Form

$$f(t) = \frac{a_0}{2} + \sum_{k=1}^{\infty} a_k \cos\frac{2\pi kt}{T} + \sum_{k=1}^{\infty} b_k \sin\frac{2\pi kt}{T}$$

where $a_k = \frac{2}{T}\int_0^T f(t)\cos\frac{2\pi kt}{T}dt$ and $b_k = \frac{2}{T}\int_0^T f(t)\sin\frac{2\pi kt}{T}dt$

Complex Exponential Form

$$f(t) = \sum_{n=-\infty}^{\infty} c_n e^{\frac{jn2\pi t}{T}}$$

where $c_n = \frac{1}{T}\int_0^T f(t)e^{-\frac{jn2\pi t}{T}}dt$

Fourier Transform

Forward

$$F(j\omega) = \mathscr{F}\{f(t)\} = \int_{-\infty}^{+\infty} f(t)e^{-j\omega t}dt$$

Inverse

$$f(t) = \mathscr{F}^{-1}\{F(j\omega)\} = \frac{1}{2\pi}\int_{-\infty}^{+\infty} F(j\omega)e^{j\omega t}d\omega$$

Laplace Transform

Bilateral

$$F(s) = \mathscr{L}\{f(t)\} = \int_{-\infty}^{+\infty} f(t)e^{-st}dt$$

Unilateral

$$F(s) = \mathscr{L}[f(t)] = \int_{0}^{+\infty} f(t)e^{-st}dt$$

Inverse

$$f(t) = \mathscr{L}^{-1}[F(s)] = \frac{1}{j2\pi}\int_{\sigma-j\infty}^{\sigma+j\infty} F(s)e^{st}ds$$

Discrete Fourier Transform (DFT)

$$X_k = \sum_{n=0}^{N-1} x_n \cdot e^{-j2kn\pi/N} = \sum_{n=0}^{N-1} x_n \cdot \left[\cos\left(\frac{2kn\pi}{N}\right) - j\cdot\sin\left(\frac{2kn\pi}{N}\right)\right]$$

$$x_n = \frac{1}{N}\sum_{k=0}^{N-1} X_k \cdot e^{j2\pi kn/N}$$

z-Transform

Bilateral (two-sided):

$$X(z) = \mathscr{Z}\{x[n]\} = \sum_{n=-\infty}^{+\infty} x[n]z^{-n}$$

Unilateral

$$X(z) = \mathscr{Z}\{x[n]\} = \sum_{n=0}^{+\infty} x[n]z^{-n}$$

Inverse

$$x[n] = \mathscr{Z}^{-1}\{X(z)\} = \frac{1}{j2\pi}\oint_C X(z)z^{n-1}dz$$

Taylor Series

$$f\left(x+\delta\right)=f\left(x\right)+\delta f'\left(x\right)+\frac{\delta^2}{2!}f''\left(x\right)+\cdots+\frac{\delta^n}{n!}f^{(n)}\left(x\right)+\cdots$$

Maclaurin Series

$$f\left(x\right)=f\left(0\right)+xf'\left(0\right)+\frac{x^2}{2!}f''\left(0\right)+\cdots+\frac{x^n}{n!}f^{(n)}\left(0\right)+\cdots$$

Quadratic Formula

If $ax^2+bx+c=0$, then

$$x_{1,2}=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

Cubic Formula

If $ax^3+bx^2+cx+d=0$,

let $p=-\frac{b}{3a}$, $q=\frac{c}{3a}$, $r=p^3+\frac{bc-3ad}{6a^2}$, and $s=\left[r^2+\left(q-p^2\right)^3\right]^{\frac{1}{2}}$

Then

$$x_{1,2,3}=\left(r+s\right)^{\frac{1}{3}}+\left(r-s\right)^{\frac{1}{3}}+p$$