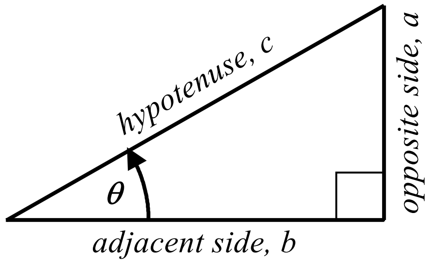
Trigonometry Basics



Note: Angles are measured counterclockwise from the horizontal axis,
and the unit of measure used here is radians.

Pythagorean Theorem

$$a^2 + b^2 = c^2$$

Definitions

$$\sin\theta = \frac{\text{opposite side}}{\text{hypotenuse}} = \frac{a}{c} \qquad \csc\theta = \frac{\text{hypotenuse}}{\text{opposite side}} = \frac{c}{a}$$

$$\cos\theta = \frac{\text{adjacent side}}{\text{hypotenuse}} = \frac{b}{c} \qquad \sec\theta = \frac{\text{hypotenuse}}{\text{adjacent side}} = \frac{c}{b}$$

$$\tan\theta = \frac{\text{opposite side}}{\text{adjacent side}} = \frac{a}{b} \qquad \cot\theta = \frac{\text{adjacent side}}{\text{opposite side}} = \frac{b}{a}$$

Functions of Some Common Angles

Angle				
Degrees	Radians	sine	cosine	tangent
0	0	0	1	0
15	$\frac{\pi}{12}$	$\frac{\sqrt{6}-\sqrt{2}}{4}$	$\frac{\sqrt{6}+\sqrt{2}}{4}$	$2-\sqrt{3}$
30	$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$

45	$\frac{\pi}{4}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	1
60	$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
75	$\frac{5\pi}{12}$	$\frac{\sqrt{6}+\sqrt{2}}{4}$	$\frac{\sqrt{6}-\sqrt{2}}{4}$	$2+\sqrt{3}$
90	$\frac{\pi}{2}$	1	0	$\pm\infty$
105	$\frac{7\pi}{12}$	$\frac{\sqrt{6}+\sqrt{2}}{4}$	$\frac{\sqrt{2}-\sqrt{6}}{4}$	$-\left(2+\sqrt{3}\right)$
120	$\frac{2\pi}{3}$	$\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	$-\sqrt{3}$
135	$\frac{3\pi}{4}$	$\frac{1}{\sqrt{2}}$	$-\frac{1}{\sqrt{2}}$	-1
150	$\frac{5\pi}{6}$	$\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{\sqrt{3}}$
165	$\frac{11\pi}{12}$	$\frac{\sqrt{6}-\sqrt{2}}{4}$	$-\frac{\sqrt{6}+\sqrt{2}}{4}$	$\sqrt{3}-2$
180	π	0	-1	0
195	$\frac{13\pi}{12}$	$\frac{\sqrt{2}-\sqrt{6}}{4}$	$-\frac{\sqrt{6}+\sqrt{2}}{4}$	$2-\sqrt{3}$
210	$\frac{7\pi}{6}$	$-\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$
225	$\frac{5\pi}{4}$	$-\frac{1}{\sqrt{2}}$	$-\frac{1}{\sqrt{2}}$	1
240	$\frac{4\pi}{3}$	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	$\sqrt{3}$
255	$\frac{17\pi}{12}$	$-\frac{\sqrt{6}+\sqrt{2}}{4}$	$\frac{\sqrt{2}-\sqrt{6}}{4}$	$2+\sqrt{3}$
270	$\frac{3\pi}{2}$	-1	0	$\mp\infty$
285	$\frac{19\pi}{12}$	$-\frac{\sqrt{6}+\sqrt{2}}{4}$	$\frac{\sqrt{6}-\sqrt{2}}{4}$	$-\left(2+\sqrt{3}\right)$
300	$\frac{5\pi}{3}$	$-\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$-\sqrt{3}$

315	$\frac{7\pi}{4}$	$-\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	-1
330	$\frac{11\pi}{6}$	$-\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$-\frac{1}{\sqrt{3}}$
345	$\frac{23\pi}{12}$	$\frac{\sqrt{2}-\sqrt{6}}{4}$	$\frac{\sqrt{6}+\sqrt{2}}{4}$	$\sqrt{3}-2$
360 (0)	2π	0	1	0

Reciprocal Relationships

$$\sin\theta = \frac{1}{\csc\theta} \qquad \csc\theta = \frac{1}{\sin\theta}$$
$$\cos\theta = \frac{1}{\sec\theta} \qquad \sec\theta = \frac{1}{\cos\theta}$$
$$\tan\theta = \frac{1}{\cot\theta} \qquad \cot\theta = \frac{1}{\tan\theta}$$

Quotient Relationships

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cot\theta = \frac{\cos\theta}{\sin\theta}$$

Periodicity

$$\sin(\theta \pm n\cdot\pi) = (-1)^n \sin\theta \qquad \csc(\theta \pm n\cdot\pi) = (-1)^n \csc\theta$$
$$\cos(\theta \pm n\cdot\pi) = (-1)^n \cos\theta \qquad \sec(\theta \pm n\cdot\pi) = (-1^n)\sec\theta$$
$$\tan(\theta \pm n\cdot\pi) = \tan\theta \qquad \cot(\theta \pm n\cdot\pi) = \cot\theta$$

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$$\sin(\theta \pm n\cdot 2\pi) = \sin\theta \qquad \csc(\theta \pm n\cdot 2\pi) = \csc\theta$$
$$\cos(\theta \pm n\cdot 2\pi) = \cos\theta \qquad \sec(\theta \pm n\cdot 2\pi) = \sec\theta$$
$$\tan(\theta \pm n\cdot 2\pi) = \tan\theta \qquad \cot(\theta \pm n\cdot 2\pi) = \cot\theta$$

Phase Between Inverses

$$\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$$
$$\tan^{-1} x + \cot^{-1} x = \frac{\pi}{2}$$
$$\sec^{-1} x + \csc^{-1} x = \frac{\pi}{2}$$

Supplement Angle Identities

$$\sin(\pi \pm \theta) = \mp \sin\theta \qquad \csc(\pi \pm \theta) = \mp \csc\theta$$
$$\cos(\pi \pm \theta) = -\cos\theta \qquad \sec(\pi \pm \theta) = -\sec\theta$$
$$\tan(\pi \pm \theta) = \pm \tan\theta \qquad \cot(\pi \pm \theta) = \pm \cot\theta$$

Symmetry

$$\sin(-\theta) = -\sin\theta \qquad \csc(-\theta) = -\csc\theta$$
$$\cos(-\theta) = \cos\theta \qquad \sec(-\theta) = \sec\theta$$
$$\tan(-\theta) = -\tan\theta \qquad \cot(-\theta) = -\cot\theta$$

Symmetry of Inverses

$$\sin^{-1}(-x) = -\sin^{-1} x$$
$$\cos^{-1}(-x) = \pi - \cos^{-1} x$$
$$\tan^{-1}(-x) = -\tan^{-1} x$$
$$\cot^{-1}(-x) = \pi - \cot^{-1} x$$
$$\sec^{-1}(-x) = \pi - \sec^{-1} x$$
$$\csc^{-1}(-x) = -\csc^{-1} x$$

Pythagorean Identities

$$\sin^2\theta + \cos^2\theta = 1$$
$$\sec^2\theta - \tan^2\theta = 1$$
$$\csc^2\theta - \cot^2\theta = 1$$

Co-Function Relationships

$$\sin\left(\theta \pm \frac{\pi}{2}\right) = \pm\cos\theta \qquad \csc\left(\theta \pm \frac{\pi}{2}\right) = \pm\sec\theta$$

$$\cos\left(\theta \pm \frac{\pi}{2}\right) = \mp\sin\theta \qquad \sec\left(\theta \pm \frac{\pi}{2}\right) = \mp\csc\theta$$

$$\tan\left(\theta \pm \frac{\pi}{2}\right) = \mp\cot\theta \qquad \cot\left(\theta \pm \frac{\pi}{2}\right) = \mp\tan\theta$$

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$$\sin\theta = \cos\left(\frac{\pi}{2} - \theta\right) \qquad \csc\theta = \sec\left(\frac{\pi}{2} - \theta\right)$$

$$\cos\theta = \sin\left(\frac{\pi}{2} - \vartheta\right) \qquad \sec\theta = \csc\left(\frac{\pi}{2} - \theta\right)$$

$$\tan\theta = \cot\left(\frac{\pi}{2} - \theta\right) \qquad \cot\theta = \tan\left(\frac{\pi}{2} - \theta\right)$$

Co-Functions Between Inverses

$$\csc^{-1} x = \sin^{-1}\left(\frac{1}{x}\right)$$

$$\sec^{-1} x = \cos^{-1}\left(\frac{1}{x}\right)$$

$$\cot^{-1} x = \tan^{-1}\left(\frac{1}{x}\right)$$

Angle Sum and Difference Relationships

$$\sin(\theta \pm \varphi) = \sin\theta\cos\varphi \pm \cos\theta\sin\varphi$$

$$\cos(\theta \pm \varphi) = \cos\theta\cos\varphi \mp \sin\theta\sin\varphi$$

$$\tan(\theta \pm \varphi) = \frac{\tan\theta \pm \tan\varphi}{1 \mp \tan\theta\tan\varphi}$$

Double-Angle Relationships

$$\sin(2\theta) = 2\sin\theta\cos\theta$$

$$\cos(2\theta) = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$$

$$\tan(2\theta) = \frac{2\tan\theta}{1-\tan^2\theta}$$

Half-Angle Relationships

$$\sin^2\theta = \frac{1}{2}\left[1-\cos(2\theta)\right]$$

$$\cos^2\theta = \frac{1}{2}\left[1+\cos(2\theta)\right]$$

$$\tan^2\theta = \frac{1-\cos(2\theta)}{1+\cos(2\theta)}$$

Sum-to-Product Transformation

$$\sin\theta + \sin\varphi = 2\sin\left(\frac{\theta+\varphi}{2}\right)\cos\left(\frac{\theta-\varphi}{2}\right)$$

$$\sin\theta - \sin\varphi = 2\cos\left(\frac{\theta+\varphi}{2}\right)\sin\left(\frac{\theta-\varphi}{2}\right)$$

$$\cos\theta + \cos\varphi = 2\cos\left(\frac{\theta+\varphi}{2}\right)\cos\left(\frac{\theta-\varphi}{2}\right)$$

$$\cos\theta - \cos\varphi = -2\sin\left(\frac{\theta+\varphi}{2}\right)\sin\left(\frac{\theta-\varphi}{2}\right)$$

Product-to-Sum Transformation

$$\sin\theta\sin\varphi = \frac{1}{2}\left[\cos(\theta-\varphi) - \cos(\theta+\varphi)\right]$$

$$\cos\theta\cos\varphi = \frac{1}{2}\left[\cos(\theta-\varphi) + \cos(\theta+\varphi)\right]$$

$$\sin\theta\cos\varphi = \frac{1}{2}\left[\sin(\theta+\varphi) + \sin(\theta-\varphi)\right] \quad \cos\theta\sin\varphi = \frac{1}{2}\left[\sin(\theta+\varphi) - \sin(\theta-\varphi)\right]$$

Rectangular to Polar Conversion

$$x + jy \rightarrow r\angle\varphi, \quad \text{where } r = \sqrt{x^2+y^2} \text{ and } \varphi = \tan^{-1}\frac{y}{x}$$

Polar to Rectangular Conversion

$$r\angle\varphi \rightarrow x + jy \quad \text{where } x = r\cos\varphi \text{ and } y = r\sin\varphi$$

Sum of Sine and Cosine

(compare with **"Rectangular to Polar Comversion"** above)

$$a\sin\omega t + b\cos\omega t = c\cos\left(\omega t + \phi\right), \text{ where } c = \sqrt{a^2 + b^2}, \text{ and } \phi = -\tan^{-1}\frac{b}{a}$$

$$a\sin\omega t + b\cos\omega t = c\sin\left(\omega t + \phi\right), \text{ where } c = \sqrt{a^2 + b^2}, \text{ and } \phi = \tan^{-1}\frac{b}{a}$$

Euler's Identity

$$e^{j\theta} = \cos\theta + j\sin\theta$$

Complex Identities

$$\sin\theta = \frac{e^{j\theta} - e^{-j\theta}}{j2}$$

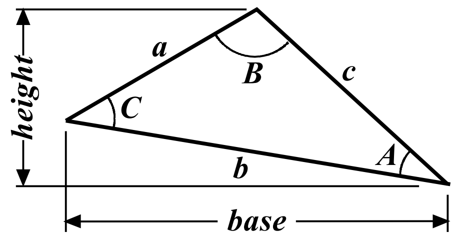
$$\cos\theta = \frac{e^{j\theta} + e^{-j\theta}}{2}$$

Miscellaneous

$$\cos mx \cos nx = \cos\left(m+n\right)x + \cos\left(m-n\right)x$$

$$\tan\left(\frac{x}{2}\right) = \frac{1-\cos x}{\sin x}$$

For *any* Triangle



Law of Sines

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Law of Cosines

$$c^2 = a^2 + b^2 - 2ab\cos C$$

Law of Tangents

$$\frac{a-b}{a+b} = \frac{\tan\left[\frac{1}{2}(A-B)\right]}{\tan\left[\frac{1}{2}(A+B)\right]}$$

Area of Triangle

$$\begin{aligned} area &= \frac{1}{2}(base)(height) \\ &= \frac{1}{2}ca\sin B \\ &= \sqrt{s(s-a)(s-b)(s-c)} \end{aligned}$$

$$\text{where } s = \frac{1}{2}(a+b+c)$$