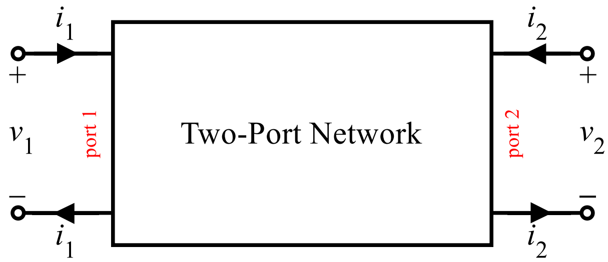
Two-Port Network Parameters



A port is a pair of terminals, such that the current entering one of those terminals leaves through the other, thus making the net current entering/leaving that port equal to zero (much like KCL). Most of the circuits of concern to us will have ***two*** ports as shown in the figure above, where an "input" signal is typically applied to the two terminals at one port and an "output" signal is then received from terminals at the other port. The parameters of the two-port network completely describe its behavior in terms of the voltage and current at the two ports; hence, knowing those parameters permits one to describe its operation when it is connected into a larger network.

Given that there are four variables associated with the two ports -- two voltages and two currents -- any two can be taken as independent, with the remaining two considered dependent and described in terms of them. Considering all potential choices, there are then six possible combinations of two independent and two dependent variables for any two-port network. Each combination can be represented by a 2×2 matrix equation as will be described shortly.

Possible Variable Assignments		
Independent	**Dependent**	**Parameter Names/Type**
$i_1,\ i_2$	$v_1,\ v_2$	Open-Circuit Impedance
$v_1,\ v_2$	$i_1,\ i_2$	Short-Circuit Admittance
$v_2,\ i_2$	$v_1,\ i_1$	Transmission
$v_1,\ i_1$	$v_2,\ i_2$	Inverse Transmission
$i_1,\ v_2$	$v_1,\ i_2$	Hybrid
$v_1,\ i_2$	$i_1,\ v_2$	Inverse Hybrid

Two-port networks are used to model electronic components, devices and systems such as transistors, OpAmps, transmission lines, filters, transformers and amplifiers. In fact, *any* linear circuit with four accessible terminals can be regarded as a two-port network provided it contains no independent sources and satisfies the port current condition mentioned above.

1. Open-Circuit Impedance Parameters (also called “z” parameters) are defined by:

$$v_1 = z_{11} i_1 + z_{12} i_2$$
$$v_2 = z_{21} i_1 + z_{22} i_2$$

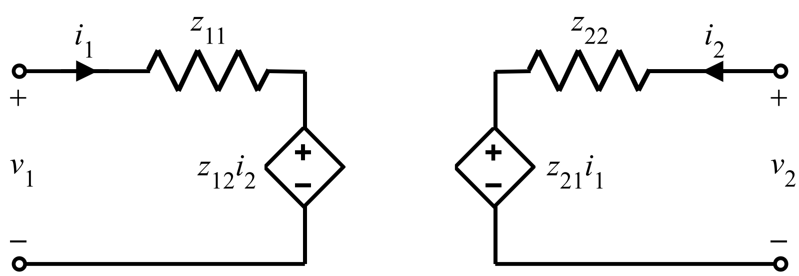
or, in matrix form,

$$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$$

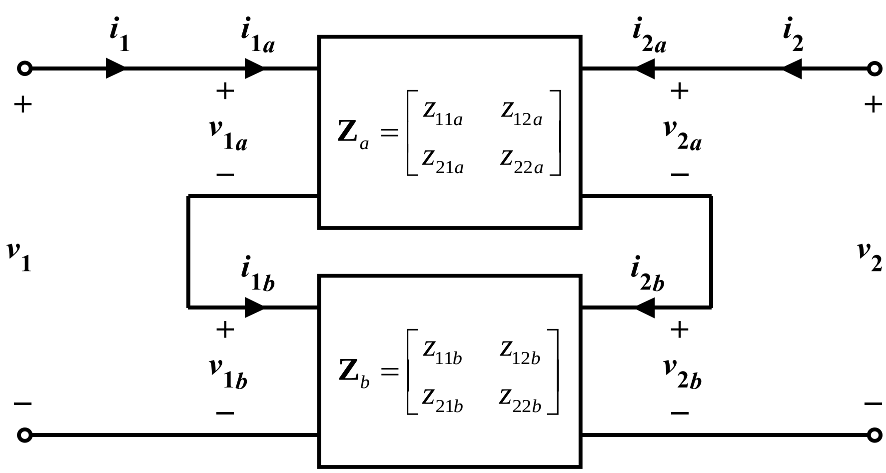
The open-circuit impedance parameters may be evaluated as follows:

$$z_{11} = \left.\frac{v_1}{i_1}\right|_{i_2=0} \qquad z_{12} = \left.\frac{v_1}{i_2}\right|_{i_1=0} \qquad z_{21} = \left.\frac{v_2}{i_1}\right|_{i_2=0} \qquad z_{22} = \left.\frac{v_2}{i_2}\right|_{i_1=0}$$

and the equivalent circuit is:



Open-circuit impedance parameters are useful for combining two two-port networks that are connected together in a **Series-Series** configuration. The impedance parameters conveniently add together, as shown below:



$$v_1 = v_{1a} + v_{1b}$$

$$i_1 = i_{1a} = i_b$$

$$v_2 = v_{2a} + v_{2b}$$

$$i_2 = i_{2a} = i_b$$

Therefore, $\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} v_{1a} + v_{1b} \\ v_{2a} + v_{2b} \end{bmatrix} = \begin{bmatrix} v_{1a} \\ v_{2a} \end{bmatrix} + \begin{bmatrix} v_{1b} \\ v_{2b} \end{bmatrix} = \mathbf{Z}_a \begin{bmatrix} i_{1a} \\ i_{2a} \end{bmatrix} + \mathbf{Z}_b \begin{bmatrix} i_{1b} \\ i_{2b} \end{bmatrix} = \mathbf{Z} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$, where $\mathbf{Z} = \mathbf{Z}_a + \mathbf{Z}_b$

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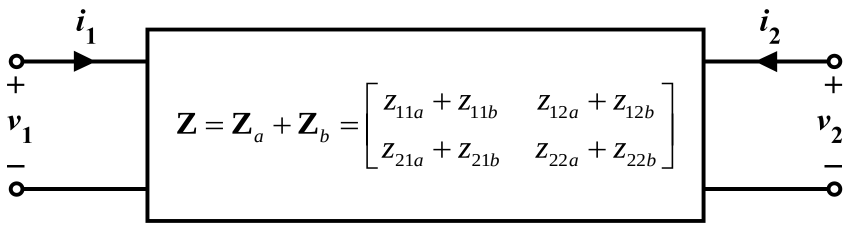


2. Short-Circuit Admittance Parameters (also called "y" parameters) are defined by:

$$i_1 = y_{11}v_1 + y_{12}v_2$$
$$i_2 = y_{21}v_1 + y_{22}v_2$$

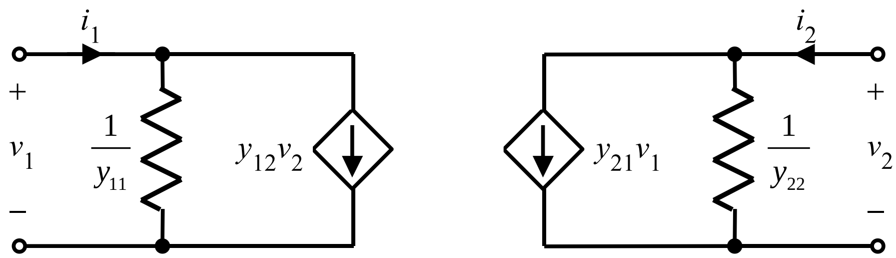
or, in matrix form,

$$\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

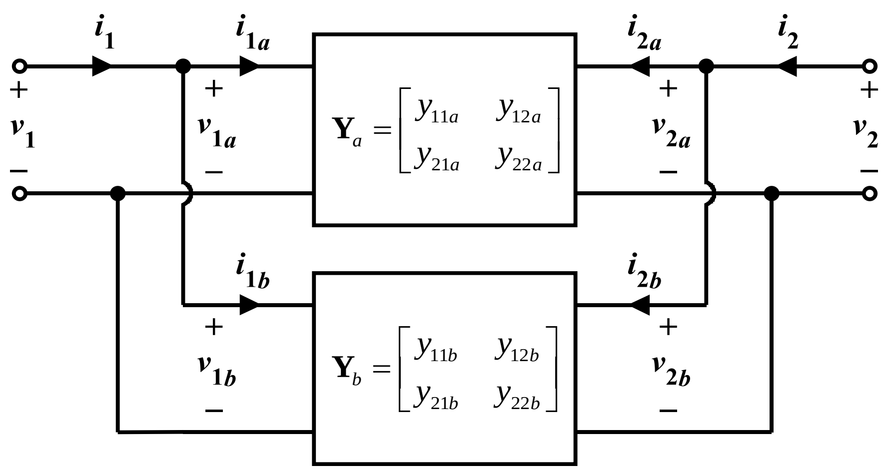
The short-circuit admittance parameters may be evaluated as follows:

$$y_{11} = \left.\frac{i_1}{v_1}\right|_{v_2=0} \qquad y_{12} = \left.\frac{i_1}{v_2}\right|_{v_1=0} \qquad y_{21} = \left.\frac{i_2}{v_1}\right|_{v_2=0} \qquad y_{22} = \left.\frac{i_2}{v_2}\right|_{v_1=0}$$

and the equivalent circuit is:



Short-circuit admittance parameters are useful for combining two two-port networks that are connected together in a **Parallel-Parallel** configuration. The admittance parameters conveniently add together, as shown below:



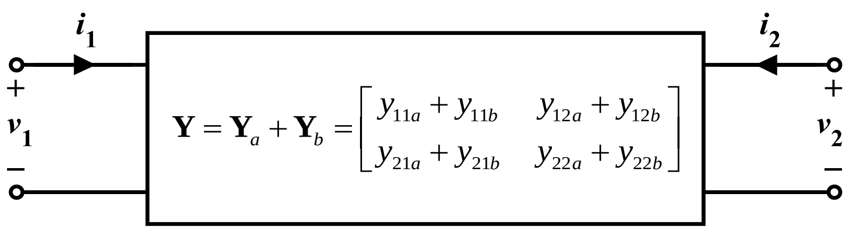
$$v_1 = v_{1a} = v_b$$

$$i_1 = i_{1a} + i_{1b}$$

$$v_2 = v_{2a} = v_{2b}$$

$$i_2 = i_{2a} + i_{2b}$$

Therefore, $\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} i_{1a} + i_{1b} \\ i_{2a} + i_{2b} \end{bmatrix} = \begin{bmatrix} i_{1a} \\ i_{2a} \end{bmatrix} + \begin{bmatrix} i_{1b} \\ i_{2b} \end{bmatrix} = \mathbf{Y}_a \begin{bmatrix} v_{1a} \\ v_{2a} \end{bmatrix} + \mathbf{Y}_b \begin{bmatrix} v_{1b} \\ v_{2b} \end{bmatrix} = \mathbf{Y} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$, where $\mathbf{Y} = \mathbf{Y}_a + \mathbf{Y}_b$.



3. Transmission Parameters (also called “a” parameters or “ABCD” parameters) are defined by:

$$v_1 = a_{11}v_2 - a_{12}i_2 \qquad \text{or, in matrix form,} \qquad \begin{bmatrix} v_1 \\ i_1 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} v_2 \\ -i_2 \end{bmatrix}$$
$$i_1 = a_{21}v_2 - a_{22}i_2$$

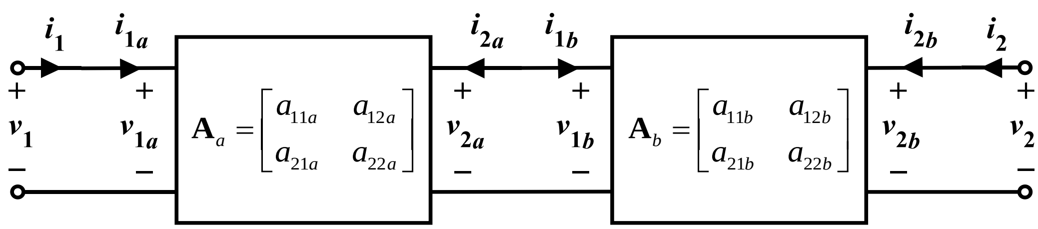
The transmission parameters may be evaluated as follows:

$$a_{11} = \left.\frac{v_1}{v_2}\right|_{i_2=0} \qquad a_{12} = -\left.\frac{v_1}{i_2}\right|_{v_2=0} \qquad a_{21} = \left.\frac{i_1}{v_2}\right|_{i_2=0} \qquad a_{22} = -\left.\frac{i_1}{i_2}\right|_{v_2=0}$$

An alternate notation is sometimes seen in older documents:

$$v_1 = Av_2 - Bi_2 \qquad \text{or, in matrix form,} \qquad \begin{bmatrix} v_1 \\ i_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} v_2 \\ -i_2 \end{bmatrix}$$
$$i_1 = Cv_2 - Di_2$$

Transmission parameters are useful for combining two two-port networks that are connected together in a **Cascade** configuration. The transmission parameters conveniently multiply together, from left to right (in order), as shown below:



$$\begin{bmatrix} v_1 \\ i_1 \end{bmatrix} = \begin{bmatrix} v_{1a} \\ i_{1a} \end{bmatrix}$$

$$\begin{bmatrix} v_{1a} \\ i_{1a} \end{bmatrix} = \mathbf{A}_a \begin{bmatrix} v_{2a} \\ -i_{2a} \end{bmatrix}$$

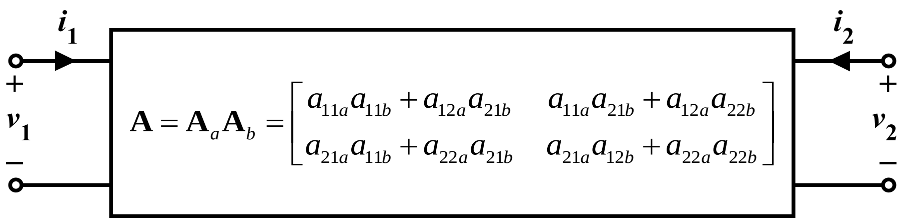
$$\begin{bmatrix} v_{2a} \\ -i_{2a} \end{bmatrix} = \begin{bmatrix} v_{1b} \\ i_{1b} \end{bmatrix}$$

$$\begin{bmatrix} v_{1b} \\ i_{1b} \end{bmatrix} = \mathbf{A}_b \begin{bmatrix} v_{2b} \\ -i_b \end{bmatrix}$$

$$\begin{bmatrix} v_{2b} \\ -i_b \end{bmatrix} = \begin{bmatrix} v_2 \\ i_2 \end{bmatrix}$$

Combining all of these, we have

$$\begin{bmatrix} v_1 \\ i_1 \end{bmatrix} = \mathbf{A}_a \mathbf{A}_b \begin{bmatrix} v_2 \\ i_2 \end{bmatrix} = \mathbf{A} \begin{bmatrix} v_2 \\ i_2 \end{bmatrix}, \text{ where } \mathbf{A} = \mathbf{A}_a \mathbf{A}_b .$$



4. Inverse Transmission Parameters (also called “b” parameters) are defined by:

$$v_2 = b_{11}v_1 - b_{12}i_1$$
$$i_2 = b_{21}v_1 - b_{22}i_1$$

or, in matrix form,

$$\begin{bmatrix} v_2 \\ i_2 \end{bmatrix} = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} \begin{bmatrix} v_1 \\ -i_1 \end{bmatrix}$$

The inverse transmission parameters may be evaluated as follows:

$$b_{11} = \left.\frac{v_2}{v_1}\right|_{i_1=0} \qquad b_{12} = -\left.\frac{v_2}{i_1}\right|_{v_1=0} \qquad b_{21} = \left.\frac{i_2}{v_1}\right|_{i_1=0} \qquad b_{22} = -\left.\frac{i_2}{i_1}\right|_{v_1=0}$$

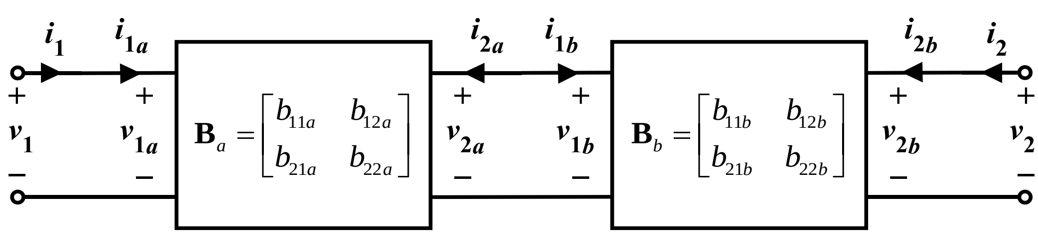
An alternate notation is sometimes seen in older documents:

$$v_2 = A'v_1 - B'i_1$$
$$i_2 = C'v_1 - D'i_1$$

or, in matrix form,

$$\begin{bmatrix} v_2 \\ i_2 \end{bmatrix} = \begin{bmatrix} A' & B' \\ C' & D' \end{bmatrix} \begin{bmatrix} v_1 \\ -i_1 \end{bmatrix}$$

Inverse transmission parameters are useful for combining two two-port networks that are connected together in a **Cascade** configuration. The inverse transmission parameters conveniently multiply together, from right to left (in reverse order), as shown below:



$$\begin{bmatrix} v_2 \\ i_2 \end{bmatrix} = \begin{bmatrix} v_{2b} \\ i_{2b} \end{bmatrix}$$

$$\begin{bmatrix} v_{2b} \\ i_{2b} \end{bmatrix} = \mathbf{B}_b \begin{bmatrix} v_{1b} \\ -i_{1b} \end{bmatrix}$$

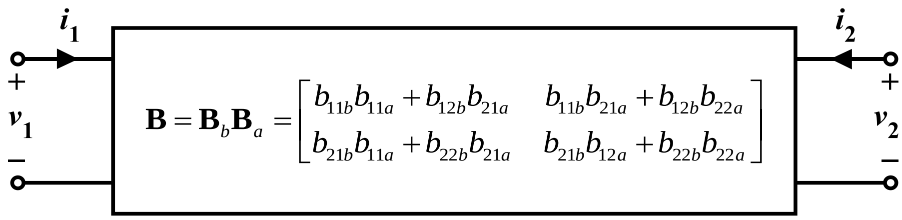
$$\begin{bmatrix} v_{1b} \\ -i_{1b} \end{bmatrix} = \begin{bmatrix} v_{2a} \\ i_{2a} \end{bmatrix}$$

$$\begin{bmatrix} v_{2a} \\ i_{2a} \end{bmatrix} = \mathbf{B}_a \begin{bmatrix} v_{1a} \\ -i_{1a} \end{bmatrix}$$

$$\begin{bmatrix} v_{1a} \\ -i_{1a} \end{bmatrix} = \begin{bmatrix} v_1 \\ -i_1 \end{bmatrix}$$

Combining all of these, we have

$$\begin{bmatrix} v_2 \\ i_2 \end{bmatrix} = \mathbf{B}_b\mathbf{B}_a \begin{bmatrix} v_1 \\ -i_1 \end{bmatrix} = \mathbf{B} \begin{bmatrix} v_1 \\ -i_1 \end{bmatrix}$$, where $\mathbf{B} = \mathbf{B}_b\mathbf{B}_a$.   (Important: Note the reverse order.)



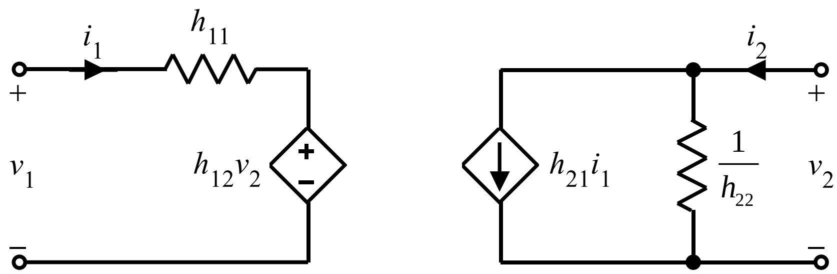
5. Hybrid Parameters (also called "h" parameters)

$$v_1 = h_{11}i_1 + h_{12}v_2 \qquad \text{or, in matrix form,} \qquad \begin{bmatrix} v_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} i_1 \\ v_2 \end{bmatrix}$$
$$i_2 = h_{21}i_1 + h_{22}v_2$$

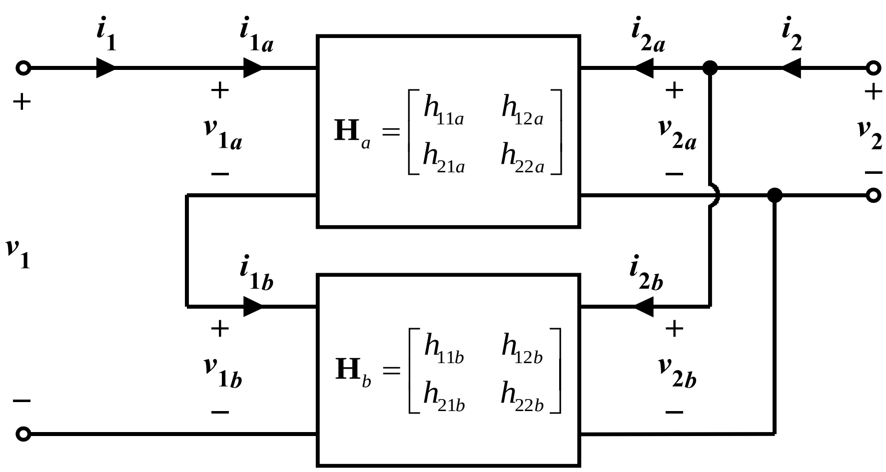
The hybrid parameters may be evaluated as follows:

$$h_{11} = \left.\frac{v_1}{i_1}\right|_{v_2=0} \qquad h_{12} = \left.\frac{v_1}{v_2}\right|_{i_1=0} \qquad h_{21} = \left.\frac{i_2}{i_1}\right|_{v_2=0} \qquad h_{22} = \left.\frac{i_2}{v_2}\right|_{i_1=0}$$

and the equivalent circuit is:



Hybrid parameters are useful for combining two two-port networks that are connected together in a **Series-Parallel** configuration. The hybrid parameters conveniently add together, as shown below:



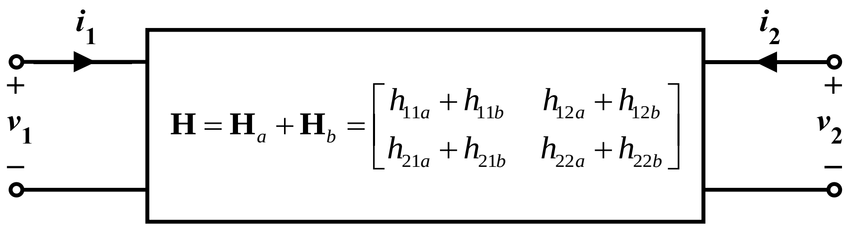
$$v_1 = v_{1a} + v_{1b}$$

$$i_1 = i_{1a} = i_{1b}$$

$$v_2 = v_{2a} = v_{2b}$$

$$i_2 = i_{2a} + i_{2b}$$

Therefore, $\begin{bmatrix} v_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} v_{1a} + v_{1b} \\ i_{2a} + i_{2b} \end{bmatrix} = \begin{bmatrix} v_{1a} \\ i_{2a} \end{bmatrix} + \begin{bmatrix} v_{1b} \\ i_b \end{bmatrix} = \mathbf{H}_a \begin{bmatrix} i_{1a} \\ v_{2a} \end{bmatrix} + \mathbf{H}_b \begin{bmatrix} i_{1b} \\ v_{2b} \end{bmatrix} = \mathbf{H} \begin{bmatrix} i_1 \\ v_2 \end{bmatrix}$, where $\mathbf{H} = \mathbf{H}_a + \mathbf{H}_b$.



6. Inverse Hybrid Parameters (also called “g” parameters)

$$i_1 = g_{11}v_1 + g_{12}i_2$$
$$v_2 = g_{21}v_1 + g_{22}i_2$$

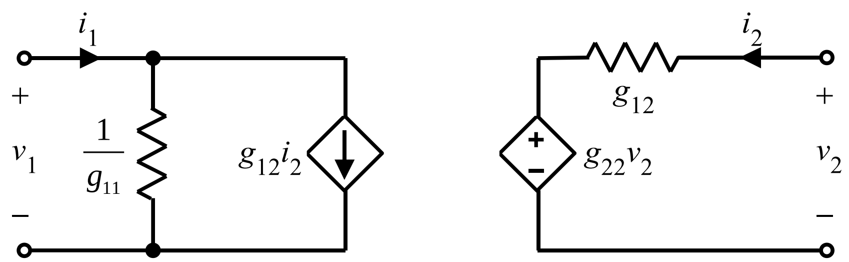
or, in matrix form,

$$\begin{bmatrix} i_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix} \begin{bmatrix} v_1 \\ i_2 \end{bmatrix}$$

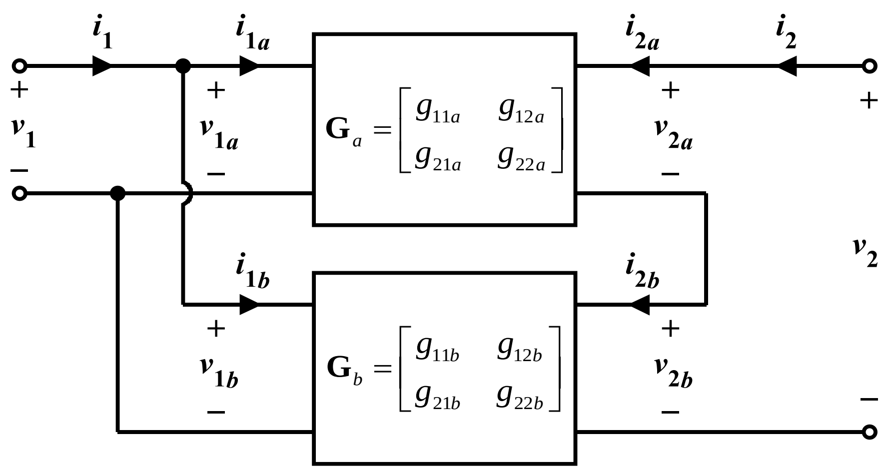
The inverse hybrid parameters may be evaluated as follows:

$$g_{11} = \left.\frac{i_1}{v_1}\right|_{i_2=0} \qquad g_{12} = \left.\frac{i_1}{i_2}\right|_{v_1=0} \qquad g_{21} = \left.\frac{v_2}{v_1}\right|_{i_2=0} \qquad g_{22} = \left.\frac{v_2}{i_2}\right|_{v_1=0}$$

and the equivalent circuit is:



Inverse hybrid parameters are useful for combining two two-port networks that are connected together in a **Parallel-Series** configuration. The inverse hybrid parameters conveniently add together, as shown below:



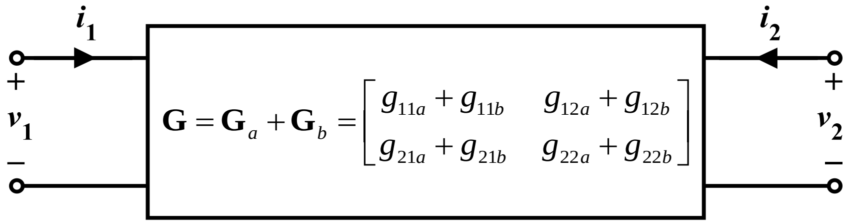
$$v_1 = v_{1a} = v_{1b}$$

$$i_1 = i_{1a} + i_{1b}$$

$$v_2 = v_{2a} + v_{2b}$$

$$i_2 = i_{2a} = i_{2b}$$

Therefore, $\begin{bmatrix} i_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} i_{1a} + i_{1b} \\ v_{2a} + v_{2b} \end{bmatrix} = \begin{bmatrix} i_{1a} \\ v_{2a} \end{bmatrix} + \begin{bmatrix} i_{1b} \\ v_{2b} \end{bmatrix} = \mathbf{G}_a \begin{bmatrix} v_{1a} \\ i_{2a} \end{bmatrix} + \mathbf{G}_b \begin{bmatrix} v_{1b} \\ i_{2b} \end{bmatrix} = \mathbf{G} \begin{bmatrix} v_1 \\ i_2 \end{bmatrix}$, where $\mathbf{G} = \mathbf{G}_a + \mathbf{G}_b$.



It is frequently necessary to convert a set of two-port parameters from one form to another, as the description given may not lend itself to convenient analysis of the network into which the two-port is to be inserted. A little algebra (not shown here) yields the transformation equations between all six of the different possible descriptions as tabulated below.

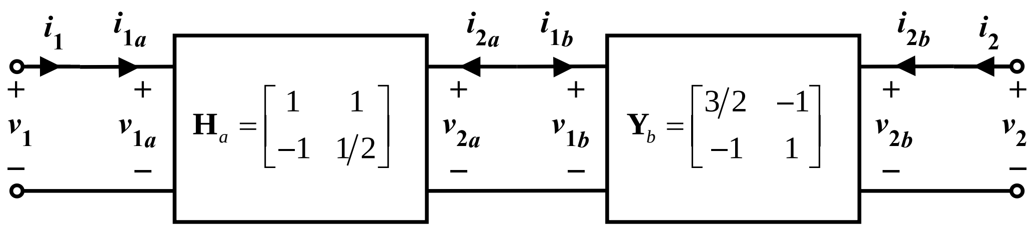
Two-Port Parameters Conversion Table

	z	y	a	b	h	g
z	$\begin{matrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{matrix}$	$\begin{matrix} \frac{y_{22}}{\Delta_y} & -\frac{y_{12}}{\Delta_y} \\ -\frac{y_{21}}{\Delta_y} & \frac{y_{11}}{\Delta_y} \end{matrix}$	$\begin{matrix} \frac{a_{11}}{a_{21}} & \frac{\Delta_a}{a_{21}} \\ \frac{1}{a_{21}} & \frac{a_{22}}{a_{21}} \end{matrix}$	$\begin{matrix} \frac{b_{22}}{b_{21}} & \frac{1}{b_{21}} \\ \frac{\Delta_b}{b_{21}} & \frac{b_{11}}{b_{21}} \end{matrix}$	$\begin{matrix} \frac{\Delta_h}{h_{22}} & \frac{h_{12}}{h_{22}} \\ -\frac{h_{21}}{h_{22}} & \frac{1}{h_{22}} \end{matrix}$	$\begin{matrix} \frac{1}{g_{11}} & -\frac{g_{12}}{g_{11}} \\ \frac{g_{21}}{g_{11}} & \frac{\Delta_g}{g_{11}} \end{matrix}$
y	$\begin{matrix} \frac{z_{22}}{\Delta_z} & -\frac{z_{12}}{\Delta_z} \\ -\frac{z_{21}}{\Delta_z} & \frac{z_{11}}{\Delta_z} \end{matrix}$	$\begin{matrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{matrix}$	$\begin{matrix} \frac{a_{22}}{a_{12}} & -\frac{\Delta_a}{a_{12}} \\ -\frac{1}{a_{12}} & \frac{a_{11}}{a_{12}} \end{matrix}$	$\begin{matrix} \frac{b_{11}}{b_{12}} & -\frac{1}{b_{12}} \\ -\frac{\Delta_b}{b_{12}} & \frac{b_{22}}{b_{12}} \end{matrix}$	$\begin{matrix} \frac{1}{h_{11}} & -\frac{h_{12}}{h_{11}} \\ \frac{h_{21}}{h_{11}} & \frac{\Delta_h}{h_{11}} \end{matrix}$	$\begin{matrix} \frac{\Delta_g}{g_{22}} & \frac{g_{12}}{g_{22}} \\ -\frac{g_{21}}{g_{22}} & \frac{1}{g_{22}} \end{matrix}$
a	$\begin{matrix} \frac{z_{11}}{z_{21}} & \frac{\Delta_z}{z_{21}} \\ \frac{1}{z_{21}} & \frac{z_{22}}{z_{21}} \end{matrix}$	$\begin{matrix} -\frac{y_{22}}{y_{21}} & -\frac{1}{y_{21}} \\ -\frac{\Delta_y}{y_{21}} & -\frac{y_{11}}{y_{21}} \end{matrix}$	$\begin{matrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{matrix}$	$\begin{matrix} \frac{b_{22}}{\Delta_b} & \frac{b_{12}}{\Delta_b} \\ \frac{b_{21}}{\Delta_b} & \frac{b_{11}}{\Delta_b} \end{matrix}$	$\begin{matrix} -\frac{\Delta_h}{h_{21}} & -\frac{h_{11}}{h_{21}} \\ -\frac{h_{22}}{h_{21}} & -\frac{1}{h_{21}} \end{matrix}$	$\begin{matrix} \frac{1}{g_{21}} & \frac{g_{22}}{g_{21}} \\ \frac{g_{11}}{g_{21}} & \frac{\Delta_g}{g_{21}} \end{matrix}$
b	$\begin{matrix} \frac{z_{22}}{z_{12}} & \frac{\Delta_z}{z_{12}} \\ \frac{1}{z_{12}} & \frac{z_{11}}{z_{12}} \end{matrix}$	$\begin{matrix} -\frac{y_{11}}{y_{12}} & -\frac{1}{y_{12}} \\ -\frac{\Delta_y}{y_{12}} & -\frac{y_{22}}{y_{12}} \end{matrix}$	$\begin{matrix} \frac{a_{22}}{\Delta_a} & \frac{a_{12}}{\Delta_a} \\ \frac{a_{21}}{\Delta_a} & \frac{a_{11}}{\Delta_a} \end{matrix}$	$\begin{matrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{matrix}$	$\begin{matrix} \frac{1}{h_{12}} & \frac{h_{11}}{h_{12}} \\ \frac{h_{22}}{h_{12}} & \frac{\Delta_h}{h_{12}} \end{matrix}$	$\begin{matrix} -\frac{\Delta_g}{g_{12}} & -\frac{g_{22}}{g_{12}} \\ -\frac{g_{11}}{g_{12}} & -\frac{1}{g_{12}} \end{matrix}$
h	$\begin{matrix} \frac{\Delta_z}{z_{22}} & \frac{z_{12}}{z_{22}} \\ -\frac{z_{21}}{z_{22}} & \frac{1}{z_{22}} \end{matrix}$	$\begin{matrix} \frac{1}{y_{11}} & -\frac{y_{12}}{y_{11}} \\ \frac{y_{21}}{y_{11}} & \frac{\Delta_y}{y_{11}} \end{matrix}$	$\begin{matrix} \frac{a_{12}}{a_{22}} & \frac{\Delta_a}{a_{22}} \\ -\frac{1}{a_{22}} & \frac{a_{21}}{a_{22}} \end{matrix}$	$\begin{matrix} \frac{b_{12}}{b_{11}} & \frac{1}{b_{11}} \\ -\frac{\Delta_b}{b_{11}} & \frac{b_{21}}{b_{11}} \end{matrix}$	$\begin{matrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{matrix}$	$\begin{matrix} \frac{g_{22}}{\Delta_g} & -\frac{g_{12}}{\Delta_g} \\ -\frac{g_{21}}{\Delta_g} & \frac{g_{11}}{\Delta_g} \end{matrix}$
g	$\begin{matrix} \frac{1}{z_{11}} & -\frac{z_{12}}{z_{11}} \\ \frac{z_{21}}{z_{11}} & \frac{\Delta_z}{z_{11}} \end{matrix}$	$\begin{matrix} \frac{\Delta_y}{y_{22}} & \frac{y_{12}}{y_{22}} \\ -\frac{y_{21}}{y_{22}} & \frac{1}{y_{22}} \end{matrix}$	$\begin{matrix} \frac{a_{21}}{a_{11}} & -\frac{\Delta_a}{a_{11}} \\ \frac{1}{a_{11}} & \frac{a_{12}}{a_{11}} \end{matrix}$	$\begin{matrix} \frac{b_{21}}{b_{22}} & -\frac{1}{b_{22}} \\ \frac{\Delta_b}{b_{22}} & \frac{b_{12}}{b_{22}} \end{matrix}$	$\begin{matrix} \frac{h_{22}}{\Delta_h} & -\frac{h_{12}}{\Delta_h} \\ -\frac{h_{21}}{\Delta_h} & \frac{h_{11}}{\Delta_h} \end{matrix}$	$\begin{matrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{matrix}$

Note that $\Delta_x = \begin{vmatrix} x_{11} & x_{12} \\ x_{21} & x_{22} \end{vmatrix} = x_{11}x_{22} - x_{12}x_{21}$ *in the above table, where* $x = z, y, a, b, h, \text{or } g$.

Example:

Suppose two networks, one whose hybrid parameters are known, and the other whose admittance parameters are known, are to be connected in a cascade configuration as shown below. Determine the impedance parameters of the combined circuit. Then sketch an equivalent circuit for the result.



Solution:

Cascaded two-ports are combined easiest in terms of their transmission or inverse transmission parameters as described above. Let's try both ways to enhance our confidence in this method of analysis by, hopefully, determining the same answer both ways.

First, convert each network's description to equivalent transmission parameters, multiply them in order, then convert the resulting description to its equivalent impedance parameters.

$$\mathbf{A}_a = \begin{bmatrix} -\dfrac{\Delta_{ha}}{h_{21a}} & -\dfrac{h_{11a}}{h_{21a}} \\ -\dfrac{h_{22a}}{h_{21a}} & -\dfrac{1}{h_{21a}} \end{bmatrix} = \begin{bmatrix} -\dfrac{(1)\left(\frac{1}{2}\right)-(1)(-1)}{(-1)} & -\dfrac{(1)}{(-1)} \\ -\dfrac{\left(\frac{1}{2}\right)}{(-1)} & -\dfrac{1}{(-1)} \end{bmatrix} = \begin{bmatrix} \dfrac{3}{2} & 1 \\ \dfrac{1}{2} & 1 \end{bmatrix}$$

$$\mathbf{A}_b = \begin{bmatrix} -\dfrac{y_{22b}}{y_{21b}} & -\dfrac{1}{y_{21b}} \\ -\dfrac{\Delta_{yb}}{y_{21b}} & -\dfrac{y_{11b}}{y_{21b}} \end{bmatrix} = \begin{bmatrix} -\dfrac{(1)}{(-1)} & -\dfrac{1}{(-1)} \\ -\dfrac{\left(\frac{3}{2}\right)(1)-(-1)(-1)}{(-1)} & -\dfrac{\left(\frac{3}{2}\right)}{(-1)} \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ \dfrac{1}{2} & \dfrac{3}{2} \end{bmatrix}$$

$$\mathbf{A} = \mathbf{A}_a\mathbf{A}_b = \begin{bmatrix} \dfrac{3}{2} & 1 \\ \dfrac{1}{2} & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ \dfrac{1}{2} & \dfrac{3}{2} \end{bmatrix} = \begin{bmatrix} \left(\frac{3}{2}\right)(1)+(1)\left(\frac{1}{2}\right) & \left(\frac{3}{2}\right)(1)+(1)\left(\frac{3}{2}\right) \\ \left(\frac{1}{2}\right)(1)+(1)\left(\frac{1}{2}\right) & \left(\frac{1}{2}\right)(1)+(1)\left(\frac{3}{2}\right) \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$$

$$\mathbf{Z}=\begin{bmatrix}\frac{a_{11}}{a_{21}} & \frac{\Delta_a}{a_{21}}\\ \frac{1}{a_{21}} & \frac{a_{22}}{a_{21}}\end{bmatrix}=\begin{bmatrix}\frac{(2)}{(1)} & \frac{(2)(2)-(3)(1)}{(1)}\\ \frac{1}{(1)} & \frac{(2)}{(1)}\end{bmatrix}=\begin{bmatrix}2 & 1\\ 1 & 2\end{bmatrix}$$

Second, convert each network's description to equivalent inverse transmission parameters, multiply them in reverse order, then convert the resulting description to its equivalent impedance parameters.

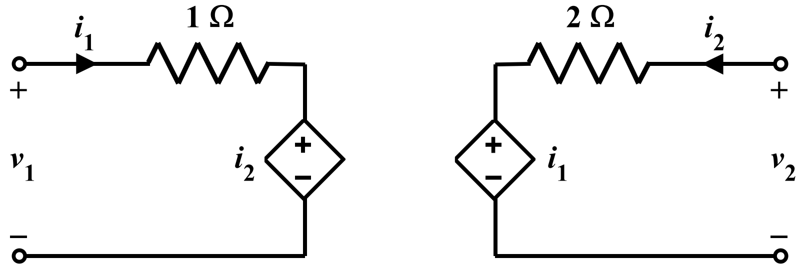
$$\mathbf{B}_a=\begin{bmatrix}\frac{1}{h_{12a}} & \frac{h_{11a}}{h_{12a}}\\ \frac{h_{22a}}{h_{12a}} & \frac{\Delta_{ha}}{h_{12a}}\end{bmatrix}=\begin{bmatrix}\frac{1}{(1)} & \frac{(1)}{(1)}\\ \frac{\left(\frac{1}{2}\right)}{(1)} & \frac{(1)\left(\frac{1}{2}\right)-(1)(-1)}{(1)}\end{bmatrix}=\begin{bmatrix}1 & 1\\ \frac{1}{2} & \frac{3}{2}\end{bmatrix}$$

$$\mathbf{B}_b=\begin{bmatrix}-\frac{y_{11b}}{y_{12b}} & -\frac{1}{y_{12b}}\\ -\frac{\Delta_{yb}}{y_{12b}} & -\frac{y_{22b}}{y_{12b}}\end{bmatrix}=\begin{bmatrix}-\frac{\left(\frac{3}{2}\right)}{(-1)} & -\frac{1}{(-1)}\\ -\frac{\left(\frac{3}{2}\right)(1)-(-1)(-1)}{(-1)} & -\frac{(1)}{(-1)}\end{bmatrix}=\begin{bmatrix}\frac{3}{2} & 1\\ \frac{1}{2} & 1\end{bmatrix}$$

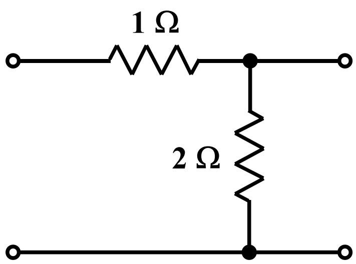
$$\mathbf{B}=\mathbf{B}_b\mathbf{B}_a=\begin{bmatrix}\frac{3}{2} & 1\\ \frac{1}{2} & 1\end{bmatrix}\begin{bmatrix}1 & 1\\ \frac{1}{2} & \frac{3}{2}\end{bmatrix}=\begin{bmatrix}\left(\frac{3}{2}\right)(1)+(1)\left(\frac{1}{2}\right) & \left(\frac{3}{2}\right)(1)+(1)\left(\frac{3}{2}\right)\\ \left(\frac{1}{2}\right)(1)+(1)\left(\frac{1}{2}\right) & \left(\frac{1}{2}\right)(1)+(1)\left(\frac{3}{2}\right)\end{bmatrix}=\begin{bmatrix}2 & 3\\ 1 & 2\end{bmatrix}$$

$$\mathbf{Z}=\begin{bmatrix}\frac{b_{22}}{b_{21}} & \frac{1}{b_{21}}\\ \frac{\Delta_b}{b_{21}} & \frac{b_{11}}{b_{21}}\end{bmatrix}=\begin{bmatrix}\frac{(2)}{(1)} & \frac{1}{(1)}\\ \frac{(2)(2)-(3)(1)}{(1)} & \frac{(2)}{(1)}\end{bmatrix}=\begin{bmatrix}2 & 1\\ 1 & 2\end{bmatrix}$$

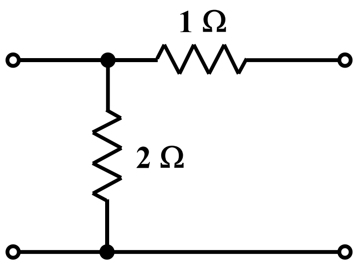
The two results are identical! The equivalent circuit then, in terms of its impedance parameters, is shown below.



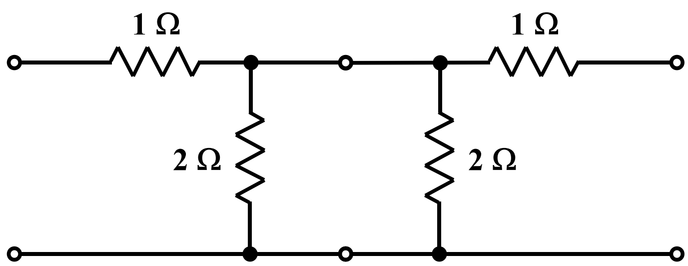
As a further check, note that two-port *a* is



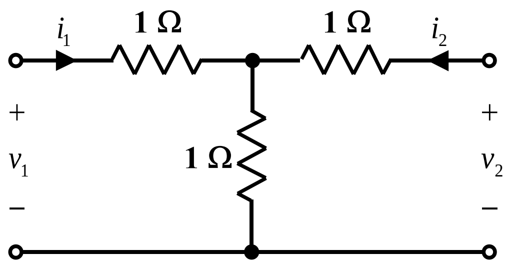
and two-port *b* is



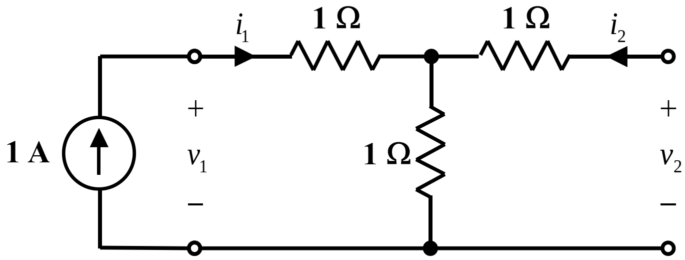
When connected in cascade, we have



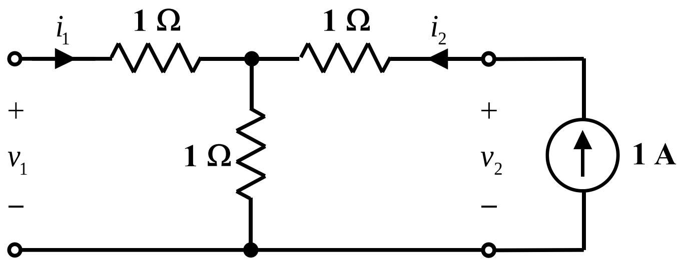
or



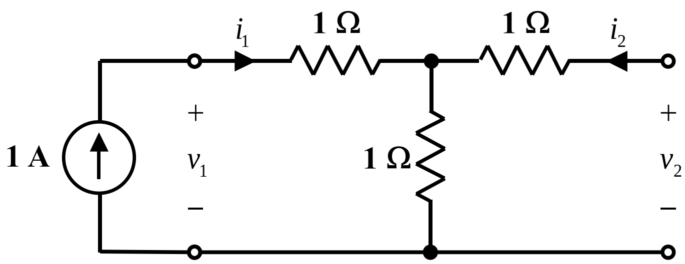
Evaluating the z-parameters of this circuit, we have



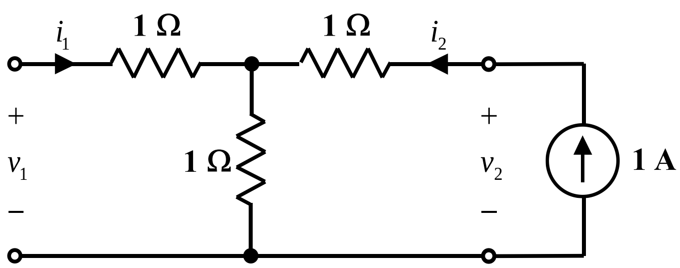
$v_1 = 2\text{ V} \;\Rightarrow\; z_{11} = 2\Omega$



$v_1 = 1\text{ V} \;\Rightarrow\; z_{12} = 1\Omega$



$v_2 = 1\text{ V} \;\Rightarrow\; z_{21} = 1\Omega$



$v_2 = 2\text{ V} \;\Rightarrow\; z_{22} = 2\Omega$

This further verifies the results determined above.